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ONE-DIMENSIONAL ELECTRO-GASDYNAMIC FLOW WITH SHOCK WAVES AND A SMALL ELECTROHYDRAULIC INTERACTION PARAMETER

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The one-dimensional flow of a unipolarly charged gas between an emitter and a collector is considered for a given discontinuous variation of velocity in the working gap (e. g. in the presence of a gasdynamic shock wave and a small parameter of electrohydraulic interaction). The effects of position and intensity of the velocity discontinuity and of the difference of electrode potentials on the flow properties are determined. It is shown that solutions yielding zero surface charges at the discontinuity can only be obtained in a limited range of variation of determining parameters. Outside that range innumerable solutions yielding nonzero surface charges are possible. A classification of solutions is made on the basis of conditions proposed in [1].

1. Let us consider a one-dimensional flow of unipolarly charged medium in the gap $0 \leqslant X \leqslant L$ between flat electrode grids for the following velocity distribution:

$$\mathbf{V} = (V_* u, 0, 0) \qquad u = \begin{cases} 1, & 0 \leqslant x < \xi \\ r, & \xi < x \leqslant 1 \end{cases}$$
$$x = X/L, \qquad V_*, r = \text{const}, \qquad r \leqslant 1$$

In a stationary one-dimensional motion of gas with a shock wave at point ξ (r is then the ratio of densities at the shock wave) such distribution of velocity obtains in the case of small parameter of electrohydraulic interaction, if the electrical forces do not affect the gasdynamic flow pattern.

The distribution of electric potential φ, electric field $E = E_x(x)$, and of bulk charge density q in the region $0 \leqslant x \leqslant 1$ without allowance for diffusion of charged particles is defined by equations

$$\varphi'' = -q, \quad q = i / (u - \varphi'), \qquad i = const, \qquad E = -\varphi' \tag{1.1}$$

where φ, E and q are dimensionless equivalents of composite characteristics $V_* L / b$, V_* / b and $\varepsilon V_* / 4\pi b L$ ($b = \text{const}$ is the mobility of charged particles and ε is the dielectric constant), respectively. Parameter i is the longitudinal component of electric current density normalized with respect to $\varepsilon V_*^2 / 4\pi b L$. A prime denotes differentiation with respect to the variable x. System (1.1) must be supplemented by boundary conditions and relationships at the discontinuity at $x = \xi$. We shall consider flows in which the potential difference $\alpha = \text{const}$ is maintained and the source of charged particles works in the state of saturation [2]

$$x = 0, \ \varphi = 0, \ E = -1; \ x = 1, \ \varphi = \alpha. \tag{1.2}$$

The dimensionless parameter α can be equal zero (flow between earthed walls), be greater than zero (external electric field directed upstream), or be less than zero (external electric field directed downstream). According to (1.1) the velocity discontinuity at $x = \xi$ must result in a discontinuity of the charge density q and, depending on the location of discontinuity ξ and its intensity $1 / r$, two cases are possible: a flow with zero surface charge at $x = \xi$ (which we shall call "continuous") or a flow with $\sigma \neq 0$ ("discontinuous" flow).

In the first case the relationships at $x = \xi$ are of the form

$$x = \xi, \ \{\varphi\} = \varphi_2 - \varphi_1 = 0, \ \{i\} = 0; \ \{E\} = 0 \tag{1.3}$$

where subscripts 1 and 2 denote, respectively, parameters up- and downstream of the shock wave front. For discontinuous solutions the conditions at $x = \xi$ can be written as

$$x = \xi, \ \{\varphi\} = 0, \ \{i\} = 0, \ E_2 + u_2 = \gamma \geqslant 0 \tag{1.4}$$

A feature of the considered problem is that for any specified ξ, r and α there exists a certain interval of variation of γ in which every point has a solution for the system (1.1), (1.2) and (1.4). The selection of physically feasible flow is made on the basis of the condition proposed in [1] according to which $\gamma = 0$.

In what follows a positively charged gas is considered. Hence

$$q > 0, \ i \geqslant 0, \ \sigma = \{E\} \geqslant 0 \tag{1.5}$$

The aim of this investigation is to determine the ranges of parameters ξ, r and α within which either continuous or discontinuous solutions exist, and to establish the singularities of distribution of electrical parameters in the working gap.

2. To solve the problem it is necessary to integrate Eqs. (1.1) in regions $x < \xi$ and $x > \xi$ and then combine the solutions at $x = \xi$. Using boundary conditions (1.2) and condition $i_1 = i_2 = i$, we obtain

$$0 \leqslant x < \xi, \ \varphi_{\mathrm{I}} = x - x\sqrt{8ix / 9} \tag{2.1}$$

$$\xi < x \leqslant 1, \ \varphi_{\mathrm{II}} = \alpha + r(x - 1) + [(r^2 + 2i + c)^{3/2} - (r^2 + 2ix + c)^{3/2}]/(3i)$$

Constants i and c (for specified ξ, r and α) are related by the equation

$$\alpha + r(\xi - 1) + \frac{1}{3i}[(r^2 + 2i + c)^{3/2} - (r^2 + 2i\xi + c)^{3/2}] - \xi + \frac{2}{3}\xi\sqrt{2i\xi} = 0 \tag{2.2}$$

which follows from condition $\{\varphi\} = 0$. If the solution is continuous, then from condition $\{E\} = 0$ with the use of (2.1) we find the closing equation

$$1-r-\sqrt{2i\xi}+\sqrt{r^2+2i\xi+c}=0 \qquad (2.3)$$

which together with (2.2) determines i and c. It can, however, be shown that the joint solution of these two equations (and by the same token prove the existence of continuous solutions) is not possible for any arbitrary ξ, r and α. It follows from (2.3) that a continuous solution is defined by the inequalities

$$1-r-\sqrt{2i\xi}\leqslant 0 \qquad (i\geqslant(1-r)^2/(2\xi)) \qquad (2.4)$$

In the case of a discontinuous solution the constants i and c are determined with the use of (2.2), (2.1) and condition $E_2+u_2=0$, which yields

$$\sqrt{i}=\frac{3}{2\sqrt{2}}\frac{[\xi+r(1-\xi)-\alpha]}{[\xi^{3/2}+(1-\xi)^{3/2}]}, \qquad c=-r^2-2i\xi \qquad (2.5)$$

From the derived formulas and the last of conditions (1.5) follows the inequality

$$E_2-E_1=\sigma(\xi,r,\alpha)=1-r-\sqrt{2i\xi}\geqslant 0 \qquad (i\leqslant(1-r)^2/(2\xi)) \qquad (2.6)$$

used as the criterion for selecting discontinuous solutions. Condition (2.6) together with (2.5) make it possible to determine the range of parameters (ξ, r) within which for fixed α continuous solutions are obtained. This range is bounded by two curves: curve *1* of set *1* (*)

$$r=\frac{2[\xi^{3/2}+(1-\xi)^{3/2}]+3\sqrt{\xi}(\alpha-\xi)}{2[\xi^{3/2}+(1-\xi)^{3/2}]+3\sqrt{\xi}(1-\xi)} \qquad (2.7)$$

and curve *2* of set *2*

$$r=(\alpha-\xi)/(1-\xi) \qquad (2.8)$$

The inequalities (2.6) and (2.4) also imply that curve (2.7) is the boundary of the region of parameters (ξ, r), in which continuous solutions exist. These two regions do not anywhere cross each other but are continuously joined along curve (2.7). It follows from (2.4) and (2.6) that this line belongs to both continuous and discontinuous modes with its points representing at the same time solutions for $\sigma=0$ and $E_2+u_2=0$. Along curve (2.8) a continuous transition takes place from the region of continuous solutions into that in which in our statement of the problem solutions do not exist at all (the "forbidden" zone). As implied by (2.5), points along curves of set *2* relate to no-load operation, since the current between electrodes is zero. In this case electric charges in the working gap are absent but according to (2.6), the surface charge at point $x=\xi$ is nonzero.

Note that in the region of discontinuous solutions the inequality

$$-1\leqslant E_1\leqslant -r \qquad (2.9)$$

which relates the electric field intensity E_1 upstream of the shock wave and the velocities on both sides of the discontinuity, is valid.

3. Depending on the dimensionless parameter α, the curves of sets *1* and *2* divide the considered region of (ξ, r) differently. For any $\alpha\leqslant 1$ curve *1*, passing through point (0, 1), crosses the square $\{0\leqslant\xi\leqslant 1,\ 0\leqslant r\leqslant 1\}$, while curve *2* crosses that square only for $\alpha\geqslant 0$. There are, thus, for $0\leqslant\alpha\leqslant 1$ three regions which corre-

*) The possibility of existence of the analytic curve (2.7) was pointed out by I. P. Semenova and A. E. Iakubenko during discussion of this problem.

spond to different solutions: region I of continuous solution, region II of discontinuous solution, and the forbidden region III. If $\alpha < 0$, the forbidden region is absent. These regions never cross each other but are continuously joined along their common boundaries.

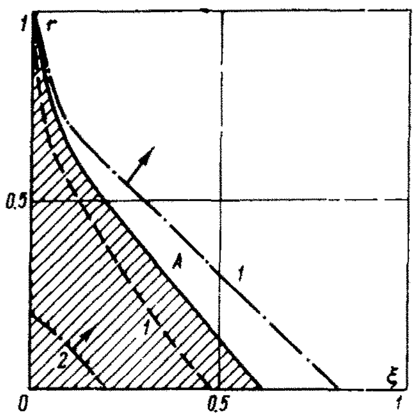

Fig. 1

The solid line A shown in Fig. 1 belongs to set 1 and relates to $\alpha = 0$. In this case the curve of set 2 degenerates into point (0, 0), hence the cross-hatched area represents region II from which it is possible to pass into region I by crossing curve A. It can be shown that for $\alpha < 0$ curves 1 move away from curve A in the direction of point (0, 0), thus reducing region II and increasing region I. The dash line in Fig. 1 relates to $\alpha = -0.2$.

Note that for $\alpha \to -\infty$ curves of set 1 tend to approach the straight line $\xi = 0$, without, however, reaching it for any finite values of α, i.e. there is always a region of discontinuous solution. This is due to the fact that, owing to the boundary condition $E(0) = -1$ at the left-hand grid, there exists for any α an arbitrarily small neighborhood of point $x = 0$, where the inequality (2.9) is valid, which indicates the existence of discontinuous solutions.

The physical meaning of this shift of lines of set 1 is explained by the shift of maximum potential toward point $x = 0$ with decreasing α at the electrode $x = 1$. This leads to the shortening of the longitudinal axis segment along which the electric field is negative. This reduces (in terms of parameter ξ) the range of possible discontinuous solutions, since their existence requires that the inequalities (2.9) be satisfied. An increase of α has the opposite effect.

It follows from formula (2.7) that for all $\alpha > 0$ curves 1 are displaced away from line A toward point (1, 1), which results in the narrowing of region I. Since for $\alpha = 1$ curve 1 becomes the straight line $r = 1$, region I degenerates into a straight line.

Formula (2.8) shows that with increasing α curve 2 moves toward point (1, 1), thus increasing region III. The dash-dot lines 1 and 2 in Fig. 1 represent the boundaries of regions for $\alpha = 0.2$, with region II lying between these. The arrows indicate the direction in which boundary curves are displaced, when α increases.

Since for $\alpha = 1$ the two boundary lines merge with the straight $r = 1$, the "rate" of displacement of curve 2 for $\alpha \to 1$ is greater than that of curve 1. Thus, the increase of parameter α results in the decrease of regions II and I. At the limit of $\alpha = 1$ the forbidden region occupies the whole of the considered square, while the remaining two regions degenerate into the straight line $r = 1$. This corresponds to no-load operation with no shock wave in the working gap.

The relative position of the three regions is shown in Fig. 2 for several values of α. Curves $1'$ and $2'$ bound region II for $\alpha = 0.2$ and curves $1''$ and $2''$ for $\alpha = 0.9$, respectively. The forbidden region and regions I lie, respectively, to the left of lines 2 and to the right of curves 1.

4. The results of computation of electrical parameters φ and q for several pairs of values of (ξ, r) and $\alpha = 0$ are shown, respectively, in Figs. 3 and 4.

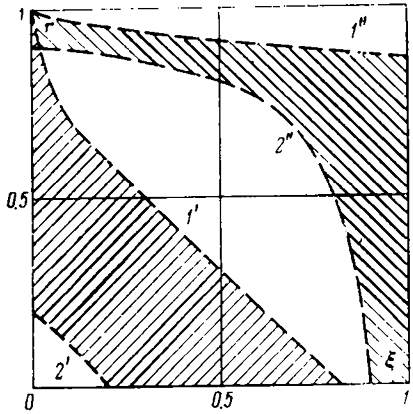

Fig. 2

It is evident that for $r = 1$ all parameters are continuous throughout the working gap. For $r < 1$ parameter q is always discontinuous at the shock wave front, and the discontinuity is the more pronounced the smaller is r or ξ, while the electric field intensity may for certain specific values of (ξ, r) be continuous.

Calculations show that the presence of a shock wave reduces the current flow between grids. The higher the intensity of the gasdynamic discontinuity at $\xi =$ const the lower the current. This can be explained by the increased accumulation of charge downstream of the shock wave with increasing shock intensity. This leads to an increase of potential in the interval $0 < x < 1$ and to a corresponding increase in absolute value of the negative electric field along the initial section of the gap. This deter-

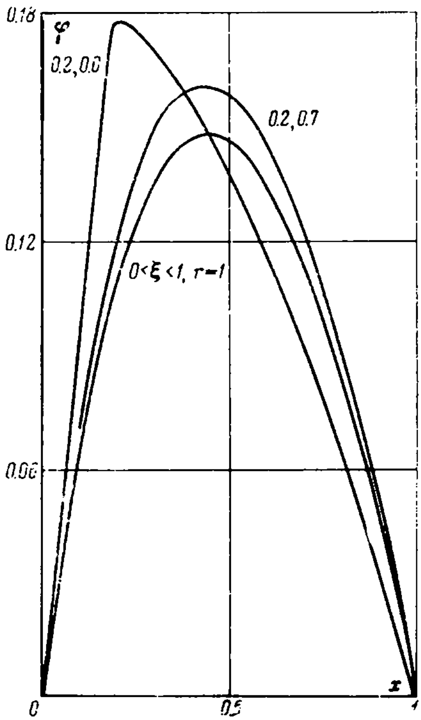

Fig. 3

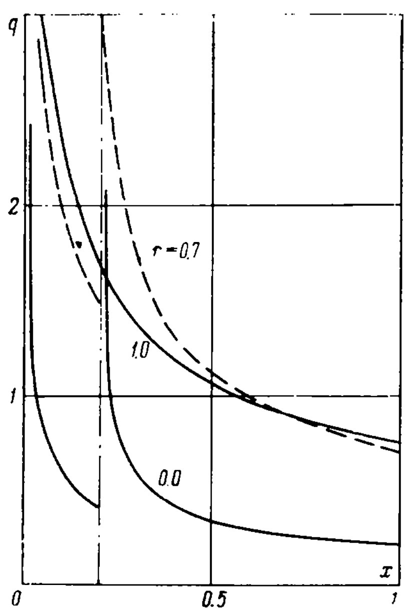

Fig. 4

mines the limit of current emission by the source. The same explanation applies to the increase of σ at the front of a gasdynamic discontinuity with increasing intensity $1 / r$

of the latter. Curves of functions $i\ (r)$ and $\sigma\ (r)$ for several fixed ξ ($\alpha = 0$) are shown in Figs. 5 and 6, respectively. Since the decrease of ξ for r = const means, in fact, the displacement of the gasdynamic discontinuity front into the region of electric fields of lower intensity, hence it results in

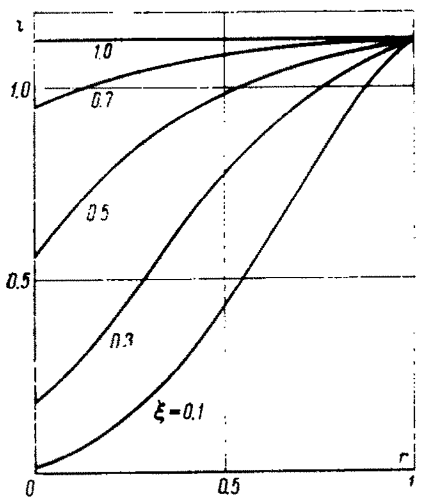

Fig. 5

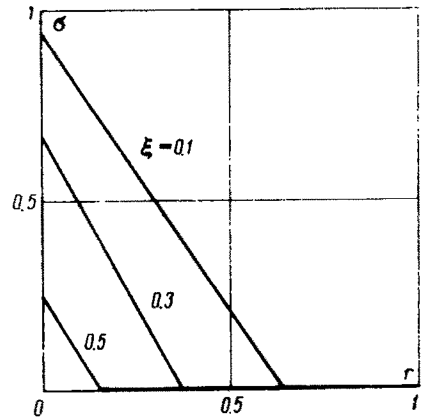

Fig. 6

increase accumulation of charge downstream of the front and, consequently, in the increase of σ and the decrease of i.

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